

40 Years Of the Eta Invariant

A Meeting to Remember V. K. Patodi

TIFR, Mumbai

Dates: 3 – 7 August, 2015

Time: 11:30 – 12:30 and 16:00 – 17:00 hrs

Speakers: P. F. Baum and E. Van Erp

The Atiyah-Singer index theorem is one of the triumphs of twentieth century mathematics. The theorem gives a topological formula for the index of any elliptic differential (or elliptic pseudo-differential) operator on a closed smooth manifold. The eta invariant of V.K. Patodi plays a central role in extending Atiyah-Singer to compact smooth manifolds with boundary.



There are two methods for proving the Atiyah-Singer theorem. The first method proves Atiyah-Singer as a corollary of Bott periodicity. The second method is the heat kernel method and uses Chern-Weil theory to directly calculate the index at the level of differential forms. The heat kernel method, when combined with the Patodi eta invariant, yields the Atiyah-Patodi-Singer theorem extending index theory to compact smooth manifolds with boundary.

INDEX THEORY AND K-HOMOLOGY

This series of five lectures will prove the Atiyah-Singer index theorem as a corollary of Bott periodicity, and then give an exposition of some further developments.

Lecture 1. Dirac Operator

The Dirac operator of R^n will be constructed. Spin-c manifolds will be introduced.

Lecture 2. Atiyah-Singer Revisited

First, some classical low-dimensional examples of the Atiyah-Singer theorem will be considered. Next, the Atiyah-Singer theorem for elliptic differential (or pseudo-differential) operators on closed smooth manifolds will be proved as a corollary of Bott periodicity.

Lecture 3. What is K-homology?

K-homology is the dual theory to K-theory. K-homology can be defined in three ways : via homotopy theory, via K-cycles, and (following Atiyah and Kasparov) via functional analysis. The lecture will give the three definitions and will explain why they are equivalent.

Lecture 4. Beyond Ellipticity

K-homology will be used to prove an index theorem for a naturally arising class of hypoelliptic (but not elliptic) differential operators

Lecture 5. The Riemann-Roch Theorem

The Grothendieck-Riemann-Roch (GRR) theorem will be reviewed. K-homology will be used to extend GRR to projective algebraic varieties which may have singularities.

ALL ARE WELCOME

A limited number of guest house rooms are available. Interested participants may write to
deanmf@math.tifr.res.in